Statistics

Lecture 9

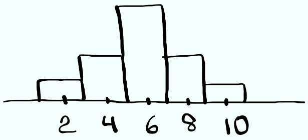
Feb 19-8:47 AM

Class QZ 4

Use the chart below

1) Draw Prob. dist. Histogram.

x	P(x)
2	.1
4	.2
6	.4
8	.2
10	.1



$\sum P(x) = 1$ ✓

x → L1, P(x) → L2

1-Var Stats

with L1 & L2

2) Find

a) $\mu = \bar{x} = 6$

b) $\sigma = \sigma_x \approx 2.191$

c) $\sigma^2 = 4.8 = \frac{24}{5}$

$\boxed{\text{VARS}}\ \boxed{\text{5: Statistics}}\ \boxed{4{:}\,\sigma x}$

$\boxed{x^2}\ \boxed{\text{Enter}}$

$\boxed{\text{Math}}\ \boxed{1{:}\blacktriangleright\text{Frac}}$

$\boxed{\text{Enter}}$

$\mu = 6$, $\sigma \approx 2$

68% Range $\mu \pm \sigma \rightarrow \boxed{4 \text{ to } 8}$

95% Range $\mu \pm 2\sigma \rightarrow \boxed{2 \text{ to } 10}$ usual Range

Oct 24-7:48 AM

Binomial Prob. Dist.

SG16

1) we have n events (trials)

They must be independent events.

2) Each event has two outcomes.

$P(\text{Success}) = p \qquad P(\text{Failure}) = q$

$p + q = 1$

$q = 1 - p$

p & q remain unchanges for all events.

3) $x \rightarrow$ # of Successes

$n - x \rightarrow$ # of failures

$$\boxed{P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}}$$

# of Combinations of having x Successes in n trials.

Oct 24-8:18 AM

Suppose $n = 5$ & $x = 2$.

Find ${}_nC_x = {}_5C_2 = \boxed{10}$

$5\ \boxed{\text{math}}\ \boxed{\rightarrow}\ \text{prb}\ \boxed{\downarrow}\ {}_nC_r\ 2\ \boxed{\text{enter}}$

5 trials, 10 ways to have 2 Successes

Suppose $n = 12$, $x = 5$

Find ${}_{12}C_5 = 792$

$12\ \boxed{\text{Math}}\ \boxed{\rightarrow}\ \text{PRB}\ \boxed{\downarrow}\ {}_nC_r\ 5\ \boxed{\text{enter}}$

12 events, there are 792 ways to have 5 Successes.

Find ${}_{50}C_5 = \boxed{2{,}118{,}760}$

CA Lotto, Pick 5 numbers from 1 to 50.

You can do that in 2,118,760 different Combinations.

Oct 24-8:24 AM

Consider a binomial Prob. dist. with
$n=5$ and $P=.6$

1) $q = 1 - P = \boxed{.4}$

2) Find P(exactly 2 Successes)

$P(x=2) = {}_5C_2 \cdot (.6)^2 (.4)^3$ $\boxed{\wedge}$ $\boxed{\div}$

$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$

$= 10(.6)^2(.4)^3 \approx \boxed{.230}$

3) P(exactly 3 Successes)

$P(x=3) = {}_5C_3 \cdot (.6)^3 (.4)^2 \approx \boxed{.346}$

Oct 24-8:32 AM

Suppose we flip a $\boxed{\text{fair coin}}$ 20 times and Success is to land tails.

1) $n=20$ 2) $P=.5$ 3) $q=.5$

4) P(land exactly 12 tails)

$P(x=12) = {}_{20}C_{12} \cdot (.5)^{12} \cdot (.5)^8$

$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$ $\approx \boxed{.120}$

5) P(land exactly 15 tails)

$P(x=15) = {}_{20}C_{15} \cdot (.5)^{15} \cdot (.5)^5 = \boxed{.015}$

6) P(lands tails 20 times)

$P(x=20) = {}_{20}C_{20} \cdot (.5)^{20} (.5)^0 = \boxed{9.5 \times 10^{-7}}$

$\underbrace{.000000}_{7 \text{ times}}95367$

Oct 24-8:39 AM

You are taking a multiple-choice quiz with 10 questions. $n=10$

Each question has 4 choices but only one correct choice. $p=\frac{1}{4}=.25$ $q=\frac{3}{4}=.75$

You are making random guesses.

P(guessing correctly exactly 5 questions)

$$P(x=5) = {}_{\underset{n}{10}}C_{\underset{x}{5}} \cdot \underset{p}{(.25)}^{\overset{x \to}{5}} \underset{q}{(.75)}^{\overset{\leftarrow n-x}{5}}$$

$= \boxed{.058}$

Using TI:

2nd VARS ↓ binompdf

$\begin{array}{ccc} n & p & x \\ (10, & .25, & 5) \end{array}$

No menu

Trials : 10

p : .25

x-value : 5

Paste Enter

Your work:

$P(x=5) = \text{binompdf}(10,.25,5)$

$= \boxed{.058}$

Oct 24-8:49 AM

Consider a binomial prob. dist. with

$n=50$ & $p=.8$

1) $q=1-p=\boxed{.2}$

2) $np=50(.8)=\boxed{40}$

3) $npq=50(.8)(.2)=\boxed{8}$

4) $\sqrt{\text{last answer}}=\sqrt{8}\approx\boxed{3}$

Let x be # of successes

5) $P(x=42)=\text{binompdf}(\overset{n}{50},\overset{p}{.8},\overset{x}{42})=\boxed{.117}$

2nd VARS ↓↓...

6) $P(x=45)=\text{binompdf}(50,.8,45)=\boxed{.030}$

Your Work

Oct 24-9:00 AM

Consider a binomial Prob. dist. with $n=40$ and $P=.6$.

1) $q=1-P=\boxed{.4}$

2) $np=40(.6)=\boxed{24}$

3) $npq=40(.6)(.4)=\boxed{9.6}$

4) $\sqrt{\text{Last answer}}=\sqrt{9.6}\approx 3$

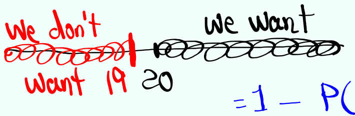
Let x be # of Successes,

5) $P(x=30)=\text{binompdf}(40,.6,30)=\boxed{.020}$

6) $P(x\le 30)=P(X=30)+P(X=29)+P(X=28)+\cdots+P(X=0)$

$=\text{binomcdf}(40,.6,30)=\boxed{.984}$

7) $P(x\ge 20)=1-P(\text{we don't want})$



$=1-P(x\le 19)$

$=1-\text{binomcdf}(40,.6,19)=\boxed{.926}$

Oct 24-9:22 AM

You are making random guesses on a True-False exam with 100 questions.

Success is to guess Correctly

1) $n=100$

2) $P=.5$

3) $q=.5$

4) $np=\boxed{50}$

5) $npq=\boxed{25}$

6) $\sqrt{npq}=\sqrt{25}=\boxed{5}$

6) P(guess correctly on exactly 60 questions)

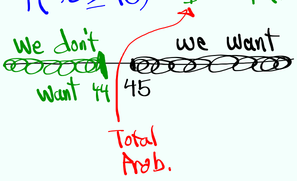
$P(x=60)=\text{binompdf}(100,.5,60)=\boxed{.011}$

7) P(guess correctly on at most 60 questions)

$P(x\le 60)=\text{binomcdf}(100,.5,60)=\boxed{.982}$

8) P(guess correctly on at least 45 questions)

$P(x\ge 45)=1-P(x\le 44)$



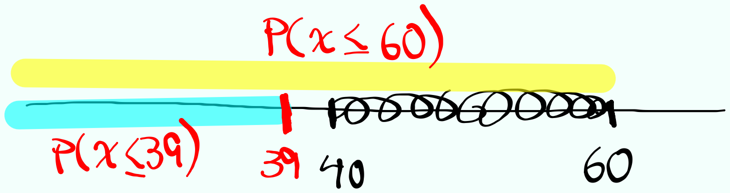
$=1-\text{binomcdf}(100,.5,44)$

$=\boxed{.864}$

Oct 24-9:31 AM

9) P(guess Correctly between 40 and 60 questions, inclusive)

$P(40 \le x \le 60) = P(x \le 60) - P(x \le 39)$



$= \text{binomcdf}(100, .5, 60) - \text{binomcdf}(100, .5, 39)$

$= \boxed{.965}$

Oct 24-9:42 AM

Consider a binomial Prob. dist. with $n = 150$ & $P = .6$

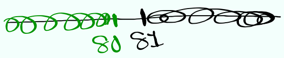
1) $q = \boxed{.4}$ 2) $np = \boxed{90}$ 3) $npq = \boxed{36}$

4) $\sqrt{npq} = \sqrt{36} = \boxed{6}$ Let x be # of Successes

5) $P(x < 100) = P(x \le 99) = \text{binomcdf}(150, .6, 99)$

$= \boxed{.944}$

6) $P(x > 80) = P(x \ge 81) = 1 - P(x \le 80)$

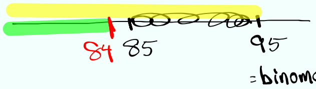


$= 1 - \text{binomcdf}(150, .6, 80)$

$= \boxed{.943}$

7) $P(85 \le x \le 95) = P(x \le 95) - P(x \le 84)$

Reduce by 1



$= \text{binomcdf}(150, .6, 95) -$
$\text{binomcdf}(150, .6, 84) =$
$\boxed{.641}$

Oct 24-9:48 AM

400 voters were randomly selected. Prob. that each voter votes Yes on Prop. 50 is .8.

1) $n = 400$ 2) $p = .8$ 3) $q = .2$

4) $np = 320$ 5) $npq = 64$ 6) $\sqrt{npq} = 8$

Let x be # of voters that vote Yes on Prop. 50.

Keep

7) $P(310 \leq x \leq 330) = P(x \leq 330) - P(x \leq 309)$

Reduce by 1

$= \text{binomcdf}(400, .8, 330) - \text{binomcdf}(400, .8, 309)$

$= \boxed{.871}$

8) $P(304 \leq x \leq 336) = P(x \leq 336) - P(x \leq 303)$

$= \text{binomcdf}(400, .8, 336) - \text{binomcdf}(400, .8, 303)$

$= \boxed{.961}$

Oct 24-10:00 AM

Working with binomial Prob. dist.

Mean	$\mu = np$
Variance	$\sigma^2 = npq$
Standard deviation	$\sigma = \sqrt{\sigma^2}$

Consider a binomial Prob. dist. with $n = 400$ & $p = .5$

1) $q = 1 - p = \boxed{.5}$ 2) $\mu = np = 400(.5) = \boxed{200}$

3) $\sigma^2 = npq = 400(.5)(.5) = \boxed{100}$

4) $\sigma = \sqrt{\sigma^2} = \sqrt{100} = \boxed{10}$

5) $\underbrace{\text{Usual Range}}_{\text{95\% Range}}$ $\mu \pm 2\sigma \Rightarrow \boxed{180 \text{ to } 220}$

6) P(# of Successes is between 180 and 220, inclusive)

$P(180 \leq x \leq 220) = \text{binomcdf}(400, .5, 220) - \text{binomcdf}(400, .5, 179) = \boxed{.960}$

Oct 24-10:23 AM

Flip a fair coin 100 times.

Success is to land tails.

1) $n = 100$ 2) $p = .5$ 3) $q = .5$

4) $\mu = 50$ 5) $\sigma^2 = 25$ 6) $\sigma = 5$

P(lands tails between 45 & 55, inclusive)

$$P(45 \le x \le 55) = P(x \le 55) - P(x \le 44)$$

$$= \text{binomcdf}(100, .5, 55) - \text{binomcdf}(100, .5, 44)$$

$$= \boxed{.729}$$

SG 16

Oct 24-10:31 AM

Working with prob. dist. with Continuous random Variable.

1) Uniform Prob. dist.

2) Standard normal Prob. dist.

3) Normal Prob. dist.

4) Central limit theorem

5) Applications

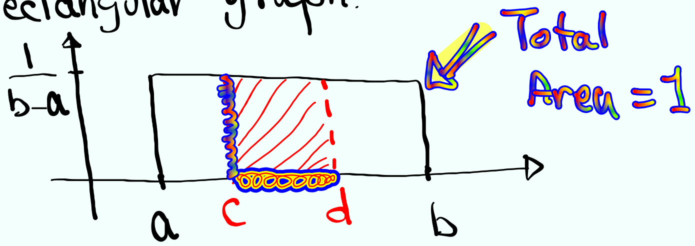
Oct 24-10:37 AM

Uniform Prob. dist.

1) $x \to$ Continuous random variable

$$P(x=c)=0$$

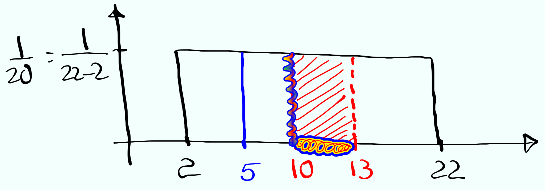
2) $a \le x \le b$, Prob. dist. has a rectangular graph.



$$P(c<x<d)=(d-c)\cdot\frac{1}{b-a}$$

Oct 24-10:40 AM

Consider uniform Prob. dist. for $2 \le x \le 22$.



$P(x=5)=\boxed{0}$ Area

↑ Line

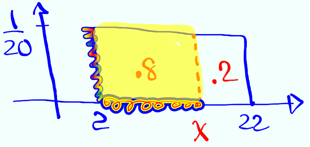
$P(10<x<13)$

$=(13-10)\cdot\frac{1}{20}=\boxed{\frac{3}{20}}$

Find $x=P_{80}$

80% below

20% above



$$(x-2)\cdot\frac{1}{20}=.8$$

$$x-2=20(.8)$$

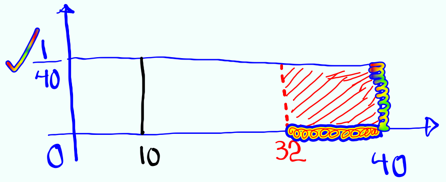
$$x=2+16=\boxed{18}$$

Oct 24-10:44 AM

Consider a uniform Prob. dist. for all values from 0 to 40.

$\frac{1}{40-0}$

$\quad b-a$

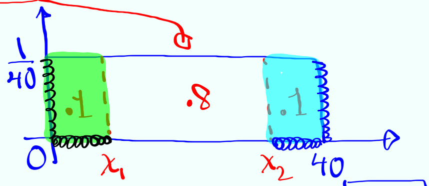


$P(x=10)=0$

$P(x>32)=(40-32)\cdot\frac{1}{40}$

$=\frac{8}{40}=\frac{1}{5}=\boxed{.2}$

Find two values that separate the middle 80% from the rest.



$(x_1-0)\cdot\frac{1}{40}=.1$

$x_1=40(.1)$

$\boxed{x_1=4}$

$(40-x_2)\cdot\frac{1}{40}=.1\ldots\ \boxed{x_2=36}$